

Mathematics:

Bar models explained.

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Today's plan:

- Brief description
- Rationale – why?
- Some positive mindset key messages
- Bar models explained in more detail – progression and examples
- Application and practice!
- Useful websites
- Year group example word problems

Mathematics:

Bar model

What is it?

In brief:

➤ ***WHAT IT IS:***

a visual aid to help understand maths relationships.

➤ ***WHAT IT IS NOT:***

a replacement for the calculations!

Bar Modelling – Why?

Apparatus / Visuals / symbols

Δ Picture



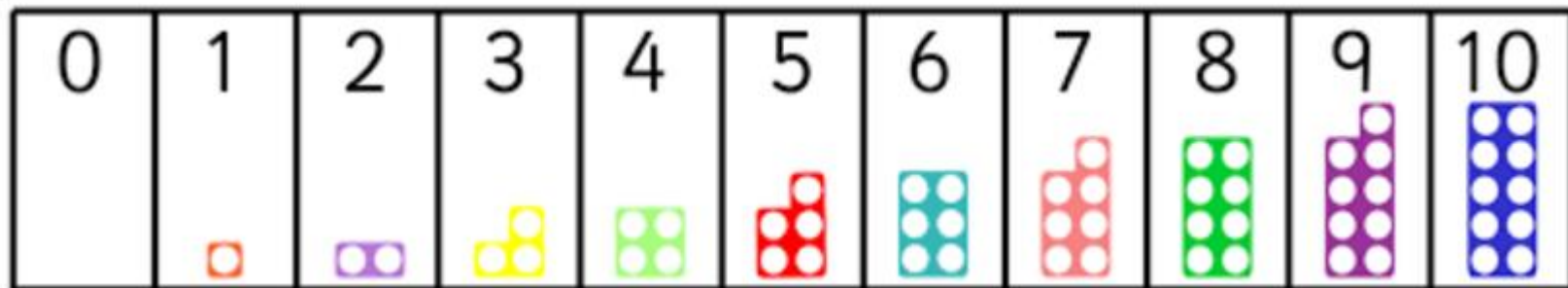
is worth
=

Creamy, delicious, yummy,
fudge ice cream, smooth,
chocolate-chip mint ice
cream, strawberry ice
cream with real chunks of
strawberry, colored sugar
sprinkles, waffle sugar
cone, sweet, wonderful,
tastes great, cold, nice to
eat, dessert, good yummy
toppings, chocolate
sprinkles, comforting,
good fun, dipping, terrific,

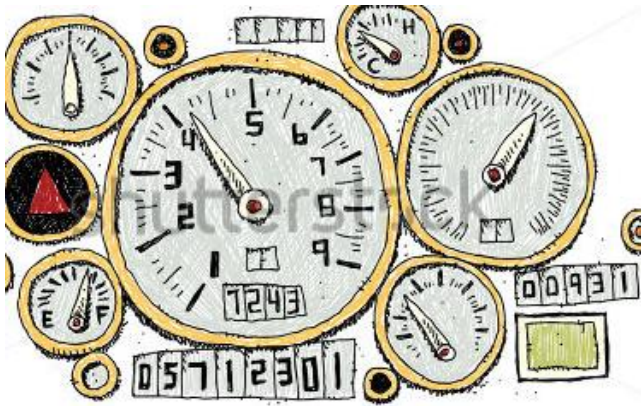
Δ thousand words.

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$$3+2=5$$



Dashboard vs engine analogy



$$5+3 = 8$$



Ways of learning



Before we start...

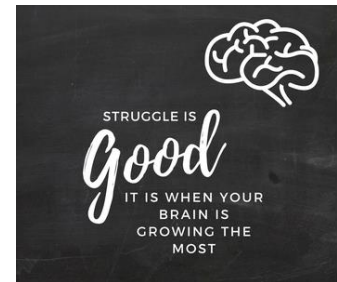
...some key messages about mindset:

"EXCELLENCE IS NOT A SKILL. IT IS AN ATTITUDE."

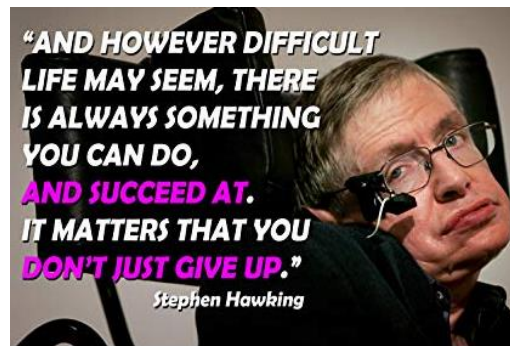
*"It is your **attitude**, not your **aptitude**, that determines your **altitude**."*

smart
is something you
become,
not something you
are.

FIXED	GROWTH
Avoid or fear challenges	Take on challenges without fear
Give up when things are tough	Keep trying in the tough times
Make excuses	Remember success takes work
Don't try so you won't fail	Learn from mistakes
Refuse to learn from mistakes	Accept feedback/criticism
Judge your classmates as smart or dumb	Celebrate the successes of your classmates
Stay where you are	Learn, improve, succeed

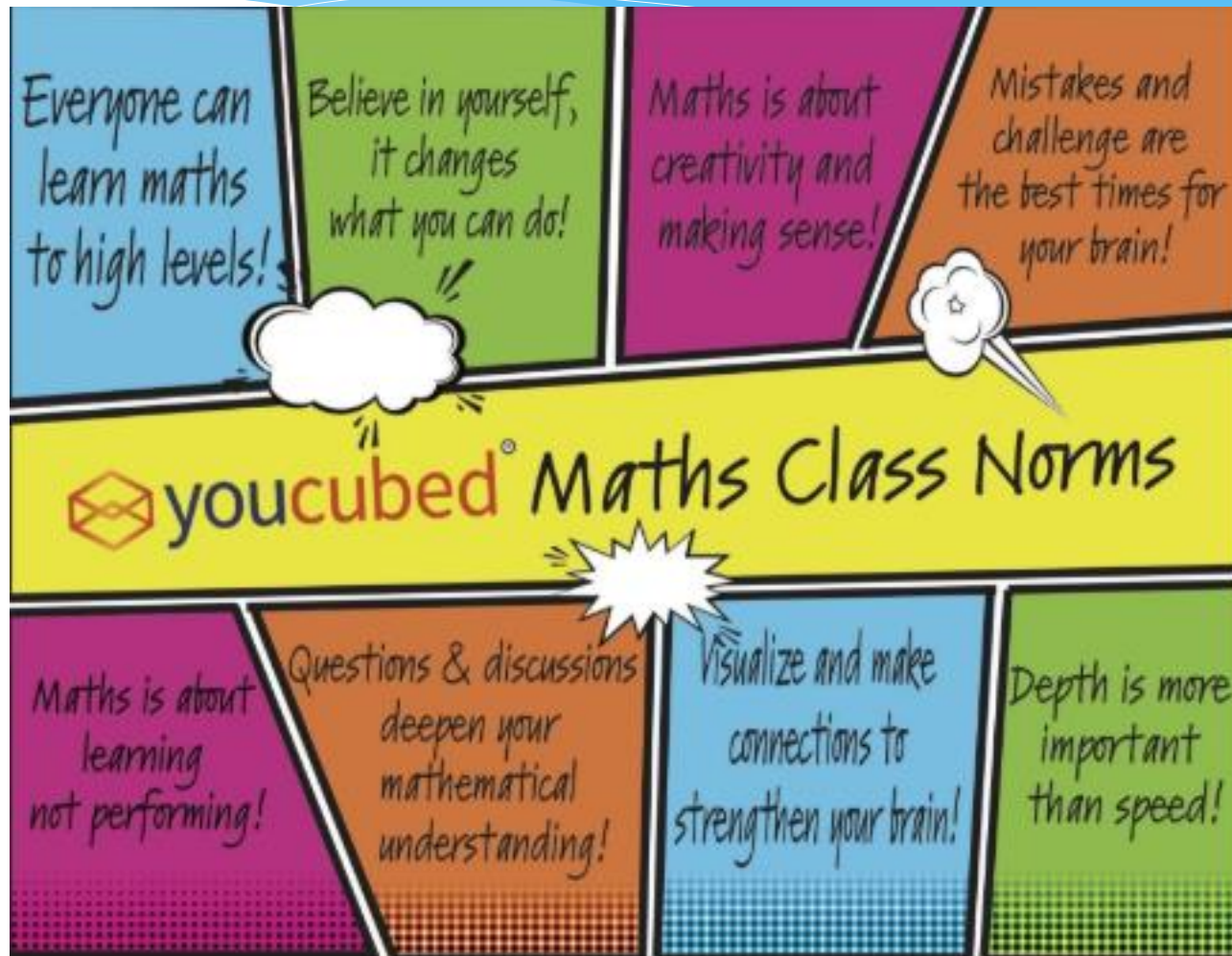


THERE is a
difference
BETWEEN
not knowing AND
NOT KNOWING YET.



ATTITUDE
is
EVERYTHING

Poster in lots of classrooms...



Teach and model self talk for children...

WHAT CAN I SAY TO MYSELF?	
INSTEAD OF...	TRY THINKING...
-I'M NOT GOOD AT THIS	-WHAT AM I MISSING?
-I'M AWESOME AT THIS	-I'M ON THE RIGHT TRACK
-I GIVE UP!	-I'LL USE SOME OF THE STRATEGIES I'VE LEARNED
-THIS IS TOO HARD	-THIS MAY TAKE SOME TIME AND EFFORT
-I CAN'T MAKE THIS ANY BETTER	-I CAN ALWAYS IMPROVE, I'LL KEEP TRYING
-I CAN'T DO MATH	-I'M GOING TO TRAIN MY BRAIN IN MATH
-I MADE A MISTAKE	-MISTAKES HELP ME IMPROVE
-I'LL NEVER BE AS SMART AS HER	-I'M GOING TO FIGURE OUT WHAT SHE DOES AND TRY IT
-IT'S GOOD ENOUGH	-IS THIS REALLY MY BEST WORK?

Praise for children which encourages growth mindset is process praise...

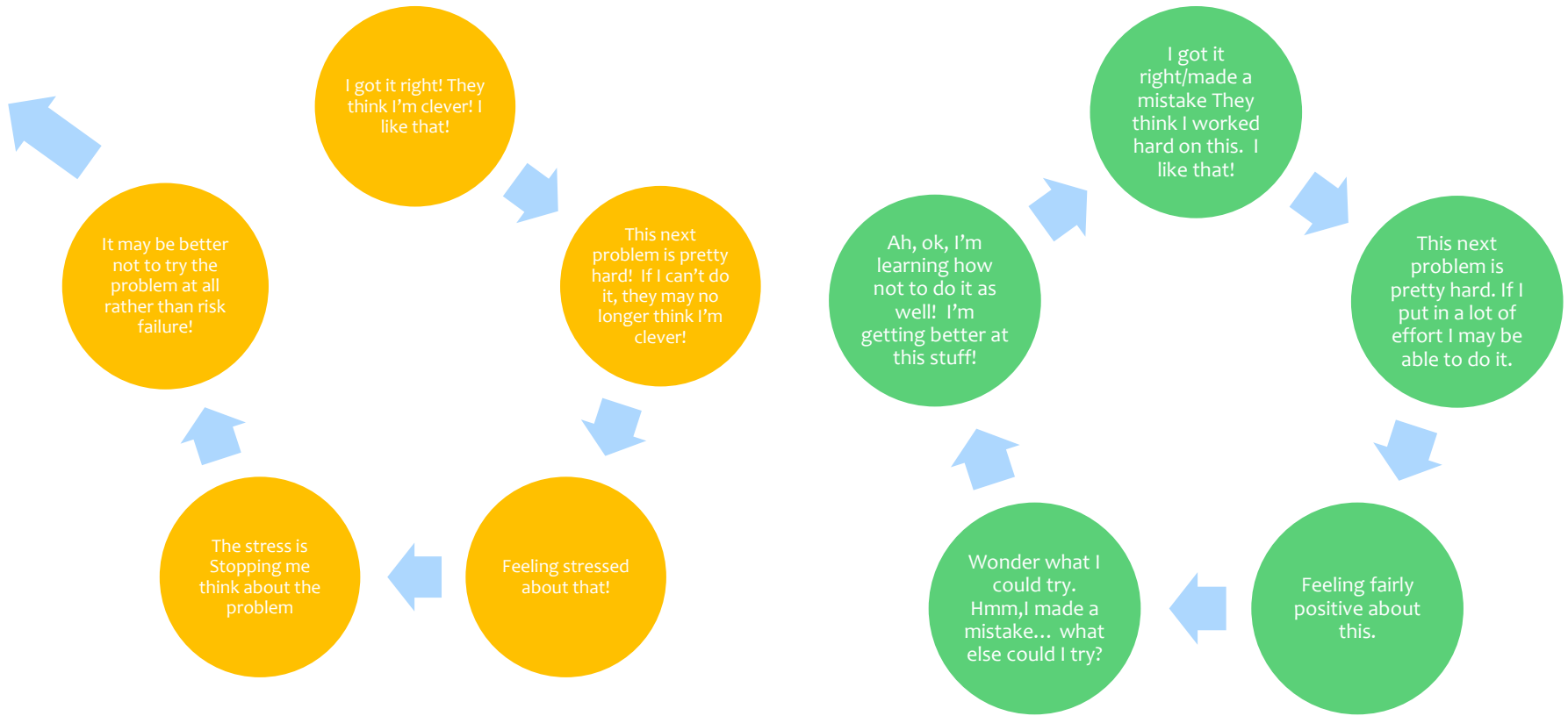
Person Praise	Process Praise
Great job! You must be smart at this.	Great job! You must have worked really hard.
See, you <i>are</i> good at English. You got an A on your last test.	You really studied for your English test and your improvement shows it.
You got it! I told you that you were smart. <i>Mrs. Winter's Bliss</i>	I like the way you tried all kinds of strategies on that math problem until you finally got it.

Beware the *negative* impact of person praise...



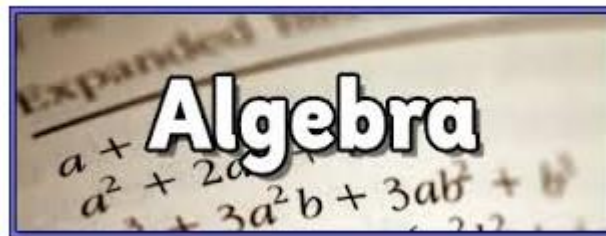
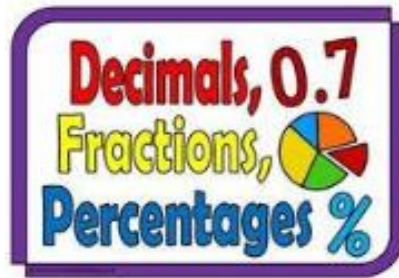
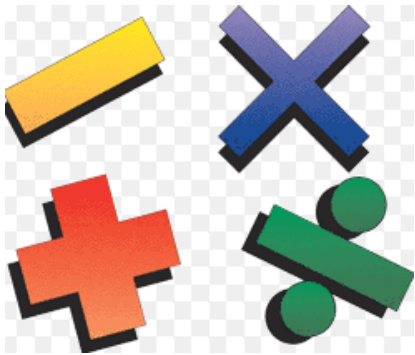
Thoughts, feelings and outcomes...

Impact of Praise on Resilience After Failure



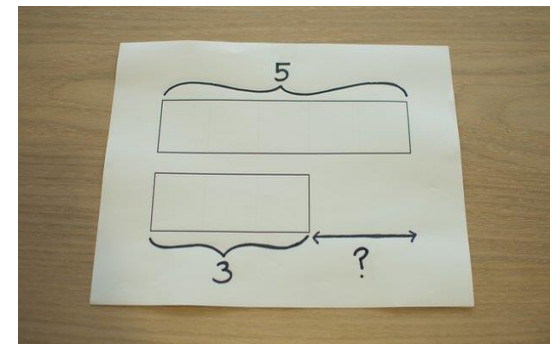
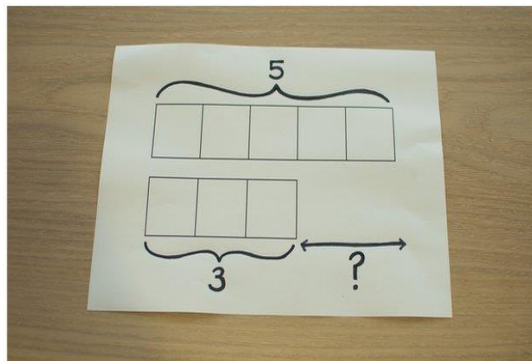
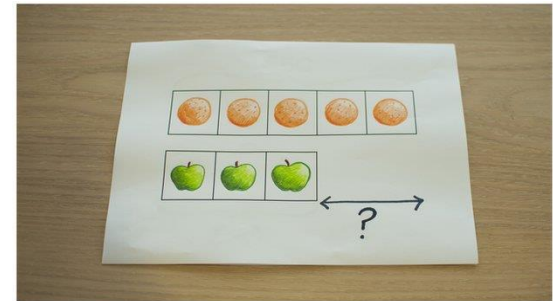
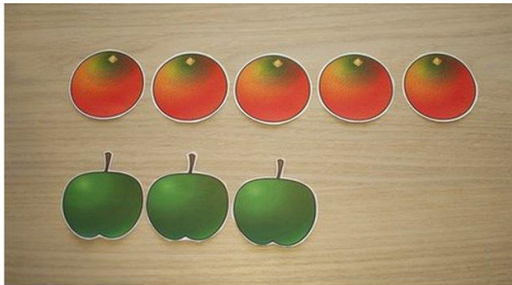
Bar modelling – where can it be used?

Almost any
mathematical
concept....

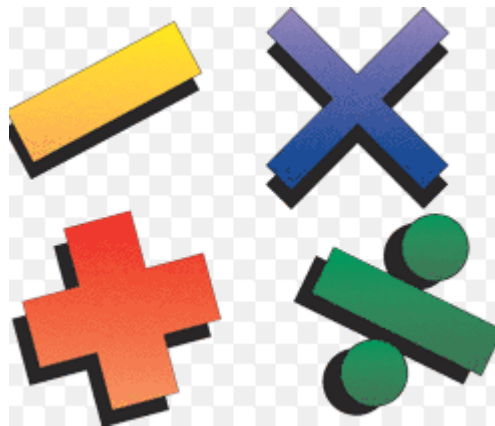


Bar modelling – so what is it?

Example word problem: Imagine you have five oranges and three apples; how many more oranges do you have than apples? *The problem here is there are a LOT of words.* So...



Looking at applications

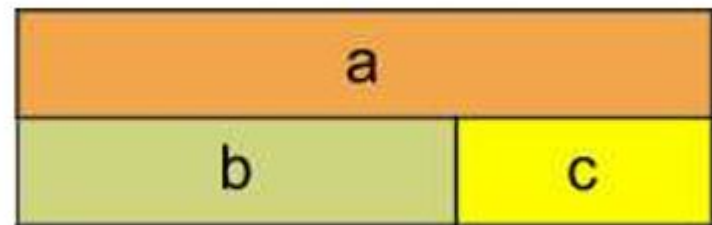


Addition and Subtraction Explanation

The bar model supports the relationship between addition and subtraction as they can both be seen within the one representation and viewed as different ways of looking at the same relationship.

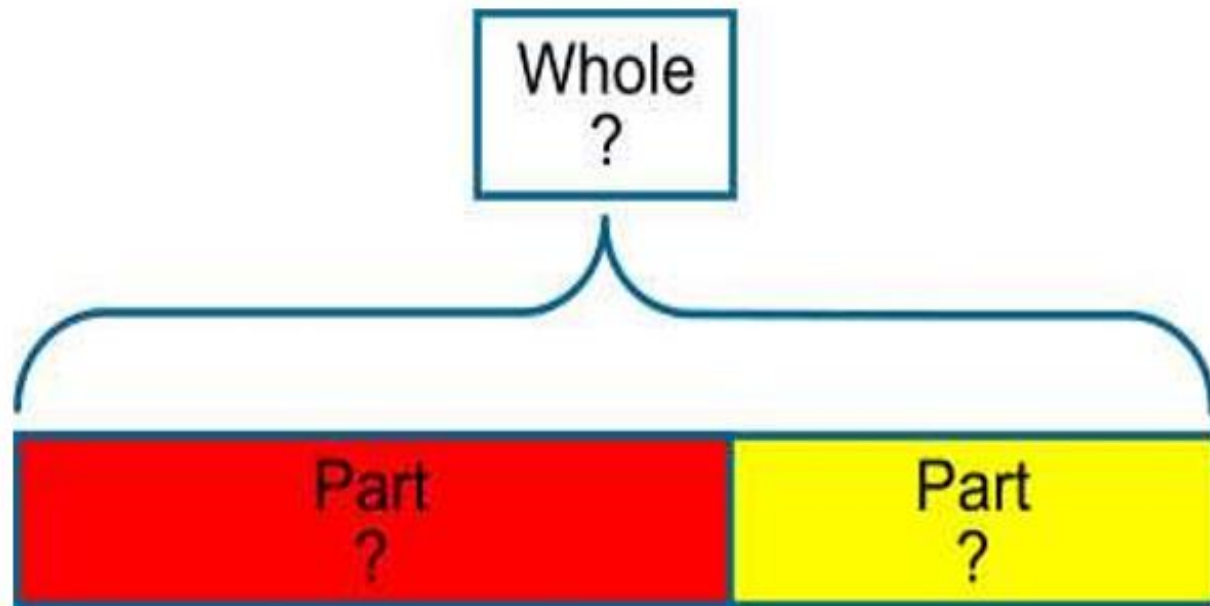
This diagram encapsulates all of the following relationships;

$$a = b + c; a + c = b; a - b = c; a - c = b$$



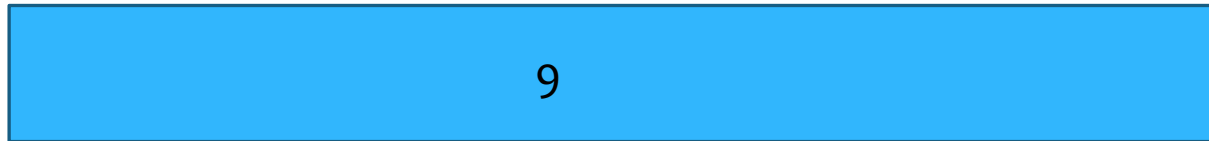
Addition and Subtraction Explanation

In problems involving addition and subtraction there are three possible unknowns as illustrated below and given the value of two of them the third can be found.



Addition and Subtraction Example

* $9 - 5 =$



?

$$5 + ? = 9$$

$$9 - ? = 5$$

$$9 = 5 + ?$$

$$5 = 9 - ?$$

$$? = 9 - 5$$

Addition and Subtraction

Example word problem

Sam had 10 red marbles and 12 blue marbles. How many marbles did he have altogether?



$$10 + 12 = 22$$

Multiplication Examples

* 5×3



Peter has 4 books

Harry has five times as many books as Peter.

How many books has Harry?



$$4 \times 5 = 20$$

Harry has 20 books

Division Examples

* $15 / 3$



When using the bar model for division it is the image of sharing rather than grouping which is highlighted in this representation.

**Mr Smith had a piece of wood that measured 36 cm.
He cut it into 6 equal pieces.
How long was each piece?**

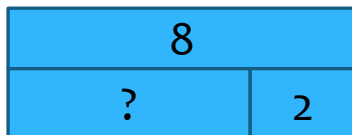
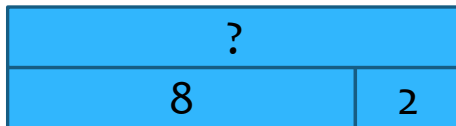


$$36 \div 6 = 6$$

Each piece is 6 cm

The four maths operations - summary

Can you match the calculations to the correct bar model, solve and explain?



* $8 + 2$

* $8 - 2$

* 8×2

* $8 \div 2$

The four maths operations - summary

Bar models to
calculations match
...solutions



$$* 8 \times 2 = 16$$



$$* 8 \div 2 = 4$$



$$* 8 + 2 = 10$$



$$* 8 - 2 = 6$$

Fractions

- * The bar model is useful for all sorts of problems involving fractions.

Find $\frac{1}{5}$ of 30

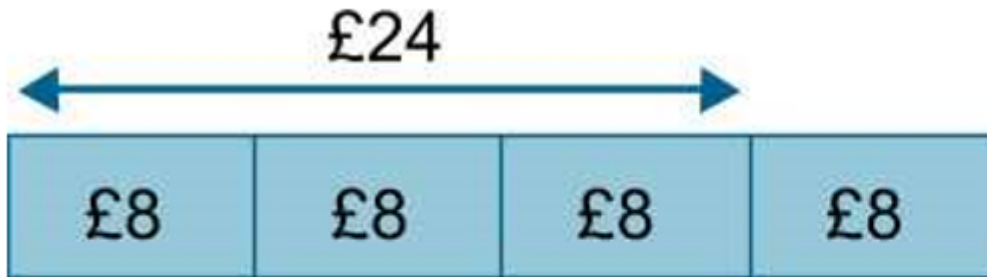


The same image can be used to find

$\frac{2}{5}$ or $\frac{3}{5}$ of 30 etc.

Fractions

A computer game is £24 in the sale. This is one quarter off its original price. How much did it cost before the sale?



The bar represents the original cost. It is divided into quarters to show the reduced cost of £24.

$£24 \div 3 = £8$, giving the value of three sections of the bar. The final section of the bar must also be £8, since it represents the same proportion as each of the other sections.

$$£8 \times 4 = £32$$

The original cost of the computer game is £32

Percentages

Problems involving percentages are solved in a similar way to those involving fractions. The key is to divide the whole into equal parts.

A computer game is reduced in a sale by 30%.
Its reduced price is £77. How much was the original price?



Dividing the bar into ten equal pieces allows us to represent 30% and keep the other pieces the same size.

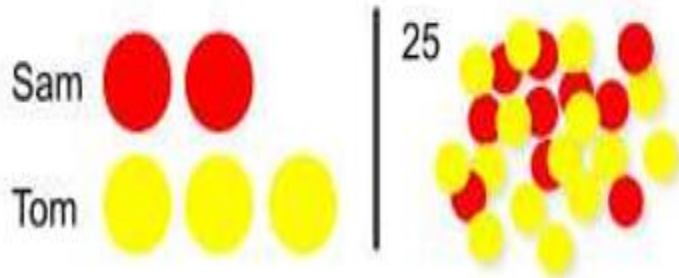
$$£77 \div 7 = £11$$

The original cost (the whole bar) is

$$£11 \times 10 = £110$$

Ratio

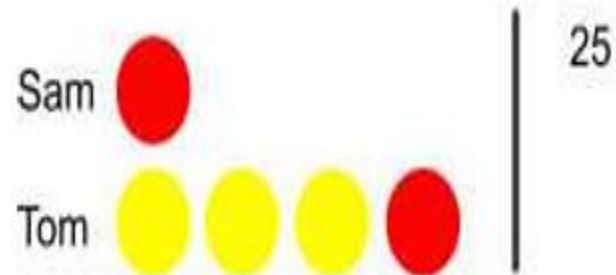
Sam and Tom have football stickers in the ratio of 2 to 3. Altogether they have 25 stickers. If Sam gives half of his stickers to Tom, how many will Tom have?



Step 1 Represent the ratio

Step 2 Recognise that if together the counters have a value of 25 then one has a value of 5

Step 3 Give half of Sam's stickers to Tom



Step 4 Multiply 5 by 4 to give the total value of Tom's stickers

Answer: $5 \times 4 = 20$



Practice.. Have a go!

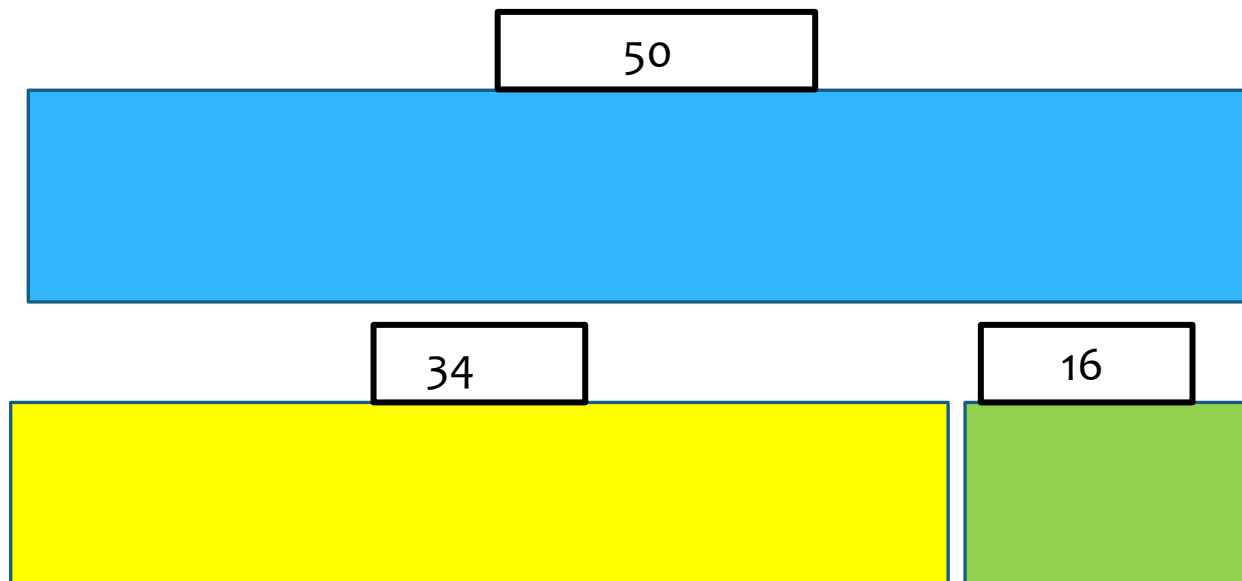
Julian had 45 toys. He gave 13 toys to his sister and 23 toys to his friend. How many toys did Julian have left?

45



More practice!

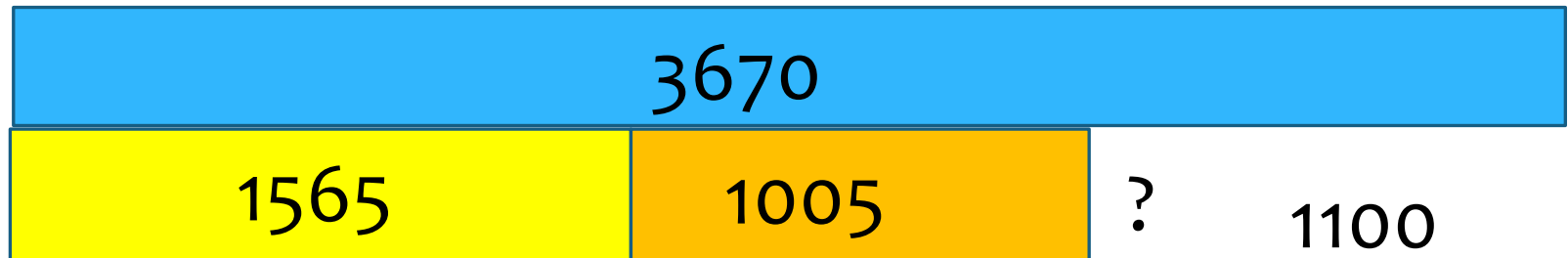
Paul has a piece of ribbon that is 50 inches long. It is 16 inches longer than Peter's ribbon. What is the length of Peter's ribbon?



Check!

Even more practice!

* There were 3670 visitors at an art exhibition. There were 1565 men and 1005 women. The rest were children. How many children went to the exhibition?



Fraction practice

Which is the larger amount, one third of £60 or one quarter of £88?

Fraction practice - solution

Which is the larger amount, one third of £60 or one quarter of £88?

$\frac{1}{3}$ of 60

$\frac{1}{3} \times 60$

$60 \div 3 = 20$

60		
20	20	20

$\frac{1}{4}$ of 88

$\frac{1}{4} \times 88$

$88 \div 4 = 22$

88			
22	22	22	22

Time to practise!

- * I hope you have seen how the **bar model** can be a valuable tool to enable children (and adults!) to **represent problems** in such a way that the **mathematical structure is revealed**.
- * This enables learners to '**see**' the problem clearly and then **recognise** the steps needed to solve it.
- * There are sets of problems available for each key stage; with your child, **have a go** at using the bar model to help identify the strategies to solve them.

Useful Sites

http://www.mathplayground.com/tb_addition/hinking_blocks_addition_subtraction.html

<http://www.thesingaporemaths.com/index.html>

<https://www.ncetm.org.uk/resources/44567>

<https://www.ncetm.org.uk/resources/26940>



Year 1 Problems

1. Ebony has 5p and Daniel has 8p. How much do they have altogether?
2. A lolly costs 6p. Amrit paid with a 10p coin. How much change does he get?
3. Michael says that $16 + 5 = 21$. Is he correct?
4. I think of a number. I subtract 5. The answer is 4. What was my number?
5. How many gloves are there altogether in 6 pairs of gloves?
6. Twelve people are split into two groups. How many are in each group?
7. Mrs Morton puts five 5p coins into her purse. How much is in her purse altogether?

Year 2 Problems

1. Dylan has 37 coloured pencils and he buys 30 more. How many does he have now?
2. Janie has 40 beads. She loses 25 of them. How many does she have left?
3. What is the difference between seventy six and thirty five?
4. I think of a number. I subtract 5. The answer is 4. What was my number?
5. Last week Ellie got £1.00 pocket money. She spent half of it. How much has she got left?
6. A tub contains 24 coins. Saj takes 5 coins. Joss takes 10 coins. How many coins are left in the tub?
7. Amelia writes the calculation below as a multiplication calculation? What might she write? $3 + 3 + 3 + 3 + 3 = 15$
8. Mr Siddique shares £18 equally between his three sons. How much does each son get?
9. Charlotte-May had to find a half of a number. Her answer was 4. What number did she start with?
10. Danny cuts his pizza into 8 equal slices. He eats half of the pizza and gives the rest to his dog, Gruff. How many pieces does Danny eat?

Year 3 Problems

1. There are 334 children at Springfield School and 75 at Holy Trinity Nursery. How many children are there altogether?
2. Gemma collected 293 badges but she gave 45 of them to her friend, Rebecca. How many badges did she have left?
3. Aiden has seven marbles and Harvey has fifteen. They decide to share them equally between them. How many do they get each?
4. Seven people each put five pens into a pot. Carmen then takes out fifteen pens. How many pens are left?
5. If you spend 61p at the corner shop, how much change do you get from £1.00?
6. If five apples cost fifty pence, how much would two apples cost?
7. Emma buys seven markers for 30p each. How much change does she get from £3.00?
8. A bookcase in the library holds 5 shelves with 46 books on each shelf. How many books are there in the bookcase altogether?
9. How many 5p stickers can Alexis buy with his 55p pocket money?
10. Which is the larger amount, one third of £60 or one quarter of £88?
11. A computer game is £24 in the sale. This is one quarter off its original price. How much did it cost before the sale?

Year 4 Problems

1. Martin has saved £6.78 and spends £4.69. How much does he have left?
2. Sally has 40 football cards. She gives $\frac{3}{5}$ of them away. How many does she give away?
3. Sally has 30 football cards. She gives $\frac{2}{5}$ of them to her friend. How many does she have left?
4. 8 children each download 59 songs to play on their iPod. How many songs do they have altogether?
5. Calculate how many fives there are in 85?
6. At the dressmakers, Debbie buys buttons weighing 3 grams each. If she has 81 grams of buttons, how many buttons does she buy?
7. Kelly buys four fifths of the shop's oranges. If the shop had 20 oranges, how many does she have?

Year 5 Problems

1. Every day for 4 days Helen scored 7.5 in a test. On the fifth day she scored 8. What was her total score?
2. I cut 60 cm from 3.3m of string and shared the rest between 3 friends. How much string did they get each?
3. How many jugs with a capacity of 250ml could you fill with 10 litres of water?
4. All the children in the school are going on a residential trip to the outdoor activity centre. They will be divided into 6 equal groups. If there are 246 children in the school how many will be in each group?
5. Robert calculated 25% of 600. What answer does he get?
6. Sam calculated 40% of 120. What answer does he get?
7. Rita worked out that $\frac{1}{6}$ of a number was 12. What was the number she started with?

Year 6 Problems

1. Three quarters of a number is 54. What is the number?
2. Which is more; $\frac{5}{9}$ of 252 or $\frac{4}{7}$ of 238?
3. There are 36 packets of biscuits. One half are chocolate, a ninth are digestive and a third are wafer biscuits. The rest are ginger nuts. How many biscuits are ginger nuts?
4. There is 20% off in a sale. How much would a track suit cost, if the normal price was £44.50?
5. There is 20% off in a sale. The reduced price of the jeans is £36. What was the original price?
6. At a dance there are 4 girls to every 3 boys. There are 63 children altogether? How many girls are there?
7. Seven in every nine packets of crisps in a box are salt and vinegar. The rest are plain. There are 63 packets of salt and vinegar crisps. How many packets of plain crisps are there?