The Catholic School of St Gregory the Great.

DEVELOPING UNDERSTADNING OF FRACTIONS, DECIMALS AND PERCENTAGES.

As a school we will use accurate and consistent vocabulary, develop variation and ensure intelligent practise when teaching fractions, decimals and percentages. This document outlines the stages of progression for teaching the above and provides examples for teachers to ensure a teaching for mastery approach.

Key Vocab:

Fraction line – Separates the denominator and numerator and means divided by.

Denominator – How many EQUAL parts the whole has been split into.

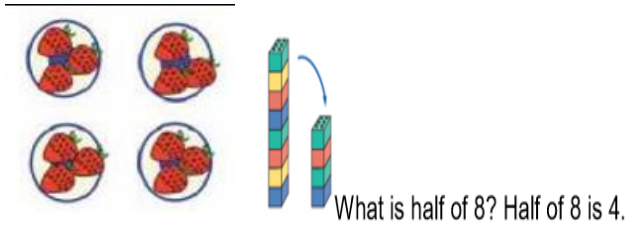
Numerator – How many equal parts we are interested in.

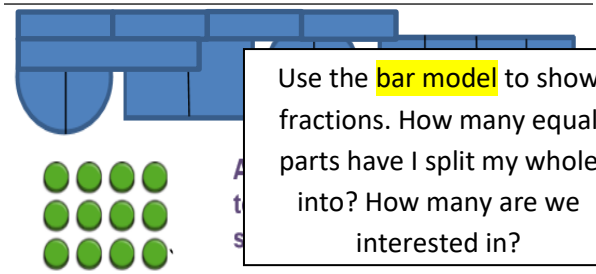
Percentage – out of 100.

Decimals, Fractions and Percentages are all parts of a whole.

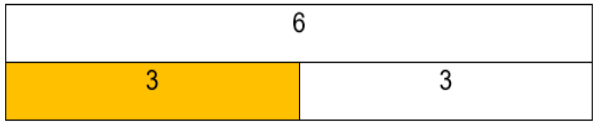
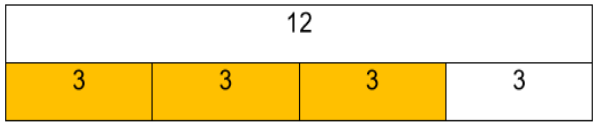
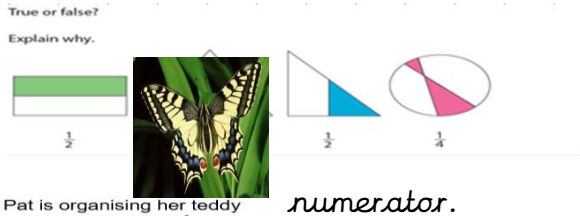
When writing a fraction we start with the fraction line, denominator and then the numerator.

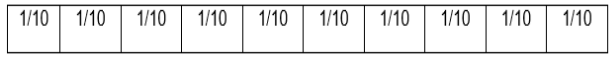
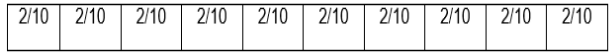
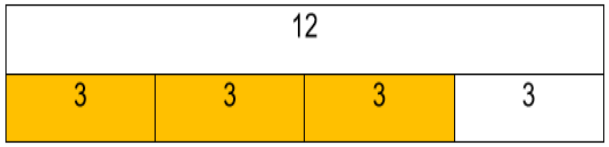
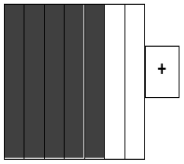
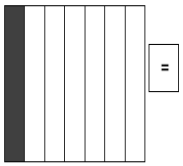
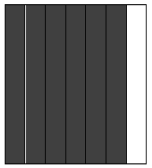
We will use real-life examples as much as possible in our teaching and teach fractions using conceptual and procedural variation.

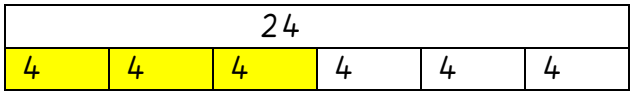
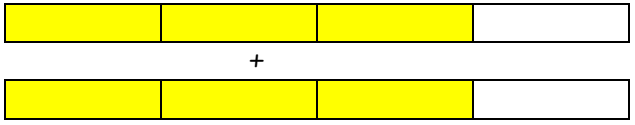
Year	NC Objectives	Examples / Misconceptions	Models and Images
EYFS	Solve problems involving halving, doubling and sharing.	I shared 10 frogs equally on 2 lily pads. Half of 10 is 5. 5 on each pad is fair. I have my whole (fruit, items). I cut it into two equal parts. I am interested in one of the two equal parts. This is ½. Can you share 9 equally? Equal – Fair. The children need to be clear on what this term means. The children find this difficult and don't always make sure they have equal parts. Use mirrors to show doubling. Use a butterfly image to help the children see that they have the 'same' again.	   Have a person in each of the bar to share the whole equally between. When doubling put the amount in one part. Show that the 'same' amount would be in the other part. And use this to work out the whole.  Use the bar model to show the total, parts of the whole, and how many you are interested

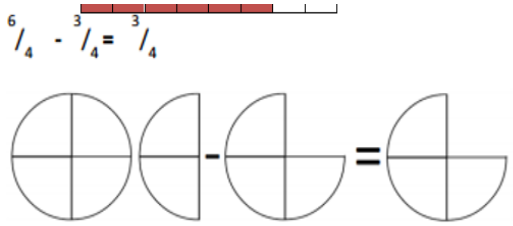
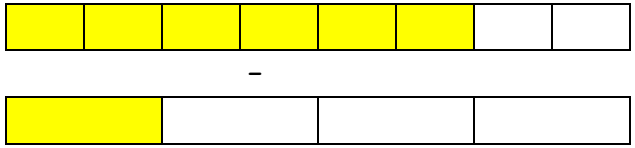
1	Recognise, find and name a half as one of two equal parts of an object, shape or quantity Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity Begin to learn sharing and grouping into equal parts.	What is a half? What is it not? What is a quarter? What is it not? I want the bigger half. Is this possible? Give me an example of a ¼... another... one that no-one else will have. My answer is ¼, what could my question be? ¼ is bigger than a ½ because it has a 4 on the bottom. Do you agree? There are 10 Smarties in a packet. If I gave you half a packet how many Smarties would I have to give? Convince me that you can split a group of 9 objects in half. • Some pupils may not understand that when splitting one whole into a fractional amount, each part must be equal • Some pupils may think that a quarter is a larger piece than a half since 4 is greater 2 • Some pupils may not appreciate that when shading a fraction of a shape, the position of the shaded section can vary.	 Use the bar model to show fractions. How many equal parts have I split my whole into? How many are we interested in? Sharing – sharing the counters among 4 people, each person gets 3. Grouping- 3 groups/ lots of 4.     Show real life-examples Start to use the language of numerator, fraction line and denominator 1 2

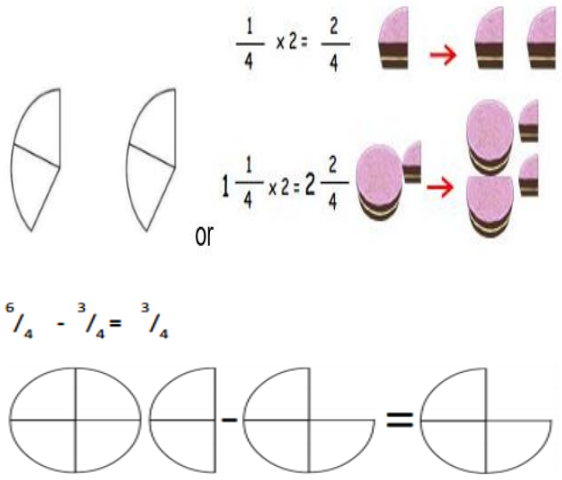
			Model writing the fraction as; Fraction line, denominator and then numerator.

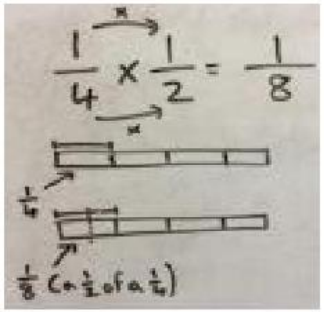
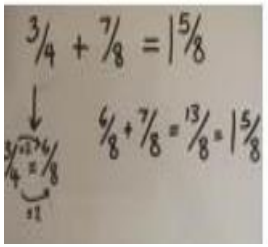
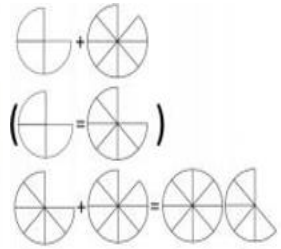
2	Recognise, find, name and write fraction 1/3, ¼, 2/4, ¾ of a length, shape, set of objects or quantity. Write simple fractions for example ½ of 6 = 3 and recognise the equivalence of 2/4 and ½.	Children use their knowledge of unit (numerator is always 1) and non-unit fractions (numerator is greater than 1) of shapes to find fractions of quantities. Use bar model, fraction wall, fraction pieces, Numicon. Vary the shape of objects used e.g. not always 'pizzas' and 'chocolate bars'. They relate this to find fractions of a length e.g. 2/4 of 1m = ¼ is greater than ½ because it has a 4 at the bottom and that is a bigger number. Do you agree? True or false: You can only find ½ of even numbers One quarter of 20 is smaller than a half of 10. • Some pupils may not understand that when splitting one whole into a fractional amount, each part must be equal • Some pupils may think that to find ¾ you split into 4 and then each of those parts into 3 • Some pupils may think that a quarter is a larger piece than a third since 4 is greater 3 • Some pupils may not appreciate that when shading a fraction of a shape, the position of the shaded section can vary	½ of 6 = 3  ¾ of 12 = 9  The children can still use dots to help equally share the total and show this in the part boxes. Model writing the fraction as; Fraction line, denominator and then  numerator. Use the bar model to show fractions. How many equal parts have I split my whole into? How many are we interested in?

3	Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one digit numbers or quantities by 10 Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators Recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators Recognise and show, using diagrams, equivalent fractions with small denominators Add and subtract fractions with the same denominator within one whole	Describe $^{1}/_{3}$ as 'there are three equal parts and I am interested in one', and $^{3}/_{4}$ as 'there are four equal parts and I am interested in three'. Also make the connection between $^{3}/_{4}$ and ' 3 of $^{1}/_{4}$ Be alert to pupils reinforcing misconceptions through language such as 'the bigger half'. To explore the equivalency of fractions make several copies of a diagram with half shaded. Show that splitting these diagrams with varying numbers of lines does not alter the fraction of the shape that is shaded. Which would you prefer ¼ of a cake? 1/3 of a cake? Or ½ of a cake? • Some pupils may think that diagrams to show fractions must always be circular • Some pupils may not acknowledge that the parts in a fraction must be equal; e.g. they talk about the 'bigger half'. • Some pupils may not appreciate the fact that a non-unit fraction is a multiple of a unit fraction. The children should be able to see that if the denominator is the same, the larger the numerator the greater the fraction. If the numerator is the same, the greater the denominator the smaller the fraction. 	Use the counting stick to demonstrate this and practise counting. 1÷ 10 = 1/10  2 ÷ 10 = 2/10  ¾ of 12 = 9  The denominator tells us how many equal parts our whole is split into (Divide by the denominator). The numerator tells us how many we are interested (multiply by the numerator). Model writing the fraction line, denominator and numerator.      

4	• Count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten • Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths • Recognise and write decimal equivalents of any number of tenths or hundredths recognise and write decimal equivalents to $^1/_4$, $^1/_2$, $^3/_4$ • add and subtract fractions with the same denominator • solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number • recognise and show, using diagrams, families of common equivalent fractions	Teachers use the language 'improper fraction' and not 'top-heavy fraction' When finding fractions of an amount pupils with denominators of at least 2 to 10 Convince me $\frac{1}{4} = 0.25$ Convince me $0.17 = \frac{17}{100}$ Show me two fractions that add together to make a whole. And another pair. And another pair. Show me one improper and one proper fraction such that the difference is less than one whole. And another pair. And another pair. Kenny thinks that ¾ of 24 is 2 because 24 ÷ 4 = 6 and 6 ÷ 3 = 2. Do you agree with Kenny? Explain your answer. Denominator – How many equal parts our whole is split into. Numerator – How many parts we are interested in. • Some pupils may read 0.25 as 'nought point twenty five' • Some pupils may think that 1/4 = 1.4 or 0.4 • Some pupils may think that you simply add/subtract the numerators and add/subtract the denominators when adding fractions. • Some pupils may think that you find the non-unit fraction of an amount by dividing by the denominator (as with unit fractions) and then dividing by the numerator. They do not make the connection that ¾ = 3 x ¼. • Some pupils may think that diagrams to show fractions must always be circular.	 Use the rows of a multiplication square to show equivalence e.g: ½, 2/4, 3/6, 4/8 2/3, 4/6. 6/9, 8/12.   Use the counting stick to count in hundredths. E.g. in steps of 1/100, 10/100 and start from different points.  ¾ + ¾ = 6/4 or 1 2/4 or 1 1/2 

			
5	Compare and order fractions whose denominators are all multiples of the same number Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents Read and write decimal numbers as fractions [for example, 0.71 = 71/100] Read, write, order and compare numbers with up to three decimal places Recognise the per cent symbol (%) and understand that per cent relates to 'number of	Describe 1/3 as 'there are three equal parts and I am interested in one', and 3/4 as 'there are four equal parts and I am interested in three'. Be alert to pupils reinforcing misconceptions through language such as 'the bigger half'. To explore the equivalency of fractions make several copies of a diagram with three-quarters shaded. Show that splitting these diagrams with varying numbers of lines does not alter the fraction of the shape that is shaded. Show me an improper fraction (mixed number). And another. Kenny thinks that $\frac{1}{4}+\frac{2}{8}=\frac{3}{12}$. Explain why Kenny is incorrect. Jenny thinks that you can only add or subtract fractions if they have the same common denominator. Do you agree with Jenny? Explain. Show me a fraction, decimal and percentage 'equivalent family' (e.g. $\frac{1}{2}$ = 50% = 0.5). And another. And another Kenny thinks that $\frac{1}{3}\times 5=\frac{5}{15}$. Do you agree with Kenny? Explain. Convince me that $2\frac{2}{3}\times 3=8$ in at least 2 different ways Show me a fraction that is equivalent to 7/10. And another …	

	parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal Solve problems involving number up to three decimal places Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $^2/_5 + {}^4/_5 = {}^6/_5 = 1\ ^1/_5$] Add and subtract fractions with the same denominator and denominators that are multiples of the same number Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams Solve problems which require knowing percentage and decimal equivalents of $^1/_2$, $^1/_4$, $^1/_5$, $^2/_5$, $^4/_5$ and those fractions with a denominator of a multiple of 10 or 25	Convince me that 6/8 is greater than 7/16 Jenny says that 0.127 is 'one hundred and twenty seven thousandths'. Kenny says that 0.127 is 'one tenth, two hundredths and seven thousandths'. Who do you agree with? Explain your reasoning. • Some pupils may read 0.234 as 'nought point two hundred and thirty four'. This leads to the common misconception that, for example, 0.400 is a number larger than 0.76 • Pupils may not make the connection that a percentage is a different way of describing a proportion Some pupils may think that equivalent fractions are found using an additive relationship rather than a multiplicative one: for example, that the fraction 4/5 is equivalent to 6/8 • Some pupils may think that you simply add the numerators and add the denominators when adding fractions. • Some pupils may think that you simply subtract the numerators and subtract the denominators when subtracting fractions. • Some pupils may think that you simply multiply both the numerator denominator when multiplying a fraction by a whole number. • Some pupils may think that you simply multiply the whole number and then the fraction when multiplying a mixed number by a whole number, e.g. $3\frac{3}{4} \times 2 = 6\frac{6}{4}$	6/8 – ¼ = 4/8 2/5 x 2 = 

	Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates		
6	Use common factors to simplify fractions; use common multiples to express fractions in the same denomination Compare and order fractions, including fractions > 1 Associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $^3/_8$] Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions	Use language carefully to avoid later confusion: when simplifying fractions, the language 'divide by 4' should not be used in place of 'divide the top and bottom by 4'. A fraction can be divided by 4, but that is not the same as cancelling a common factor of the numerator and denominator by dividing them by 4. Use of a fraction wall / bar model to visualize multiplying fractions and dividing fractions by a whole number. For example, pupils need to read calculations such as $\frac{1}{4} \times \frac{1}{2}$ as $\frac{1}{4}$ multiplied by $\frac{1}{2}$ and therefore, $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$; $\frac{4}{10} \div 2$ as $\frac{4}{10}$ divided by 2 and therefore $\frac{2}{10}$. • Show me another fraction that is equivalent to this one. And another. And another ... • Convince me that 3/8 = 0.375 • If you know that 1/10 = 0.1 = 10%, what else can you work out? • Jenny is simplifying fractions. She has the fraction 16/64. Jenny says, 'if I cancel out the sixes then 16/64 = 1/4.'. Do you agree with Jenny? Why? • Show me an 'easy' ('difficult') pair of fractions to add (subtract). And another. And another. • Kenny thinks that $\frac{7}{10} - \frac{2}{7} = \frac{5}{3} = 1\frac{2}{3}$. Do you agree with Kenny?	   A computer games is £24 In the sale. There is one quarter off its original price. How much did it cost before the sale?

	Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $^1/_4 \times {}^1/_2 = {}^1/_8$] Divide proper fractions by whole numbers [for example, $^1/_3 \div 2 = {}^1/_6$] Multiply one-digit numbers with up to two decimal places by whole numbers Solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison	• Jenny thinks that you can only multiply fractions if they have the same common denominator. Do you agree with Jenny? Explain. • Benny thinks that $\frac{4}{10} \div 2 = \frac{4}{5}$. Do you agree with Kenny? Explain. • Lenny says '20% of £60 is £3 because 60 ÷ 20 = 3'. Do you agree? • A fraction can be visualised as divisions of a shape (especially a circle) but some pupils may not recognise that these divisions must be equal in size, or that they can be divisions of any shape. • Pupils may not make the connection that a percentage is a different way of describing a proportion • Some pupils may think that simplifying a fraction just requires searching for, and removing, a factor of 2 (repeatedly) • Some pupils may think that you simply can simply add/subtract the whole number part of mixed numbers and add/subtract the fractional art of mixed numbers when adding/subtracting mixed numbers, e.g. $3\frac{1}{3} - 2\frac{1}{2} = 1\frac{-1}{6}$ • Some pupils may make multiplying fractions over complicated by applying the same process for adding and subtracting of finding common denominators. • Some pupils may think that as you divide by 10 to find 10%, you divided by 15 to find 15%, divide by 20 to find 20%, divide by 100 to find 100%, etc. • Children get confused by dividing fractions as you end up using multiplication – when we divide it, we are sharing it by more...	<table><tr><td colspan="4">24</td></tr><tr><td>8</td><td>8</td><td>8</td><td>8</td></tr></table>

Useful Resources:	**Can do Fractions – Saved in the common room. This should be the main resources that we use for teaching fractions to ensure consistency in vocabulary and teaching representations across the school.** Teaching for mastery documents NCTEM – https://www.ncetm.org.uk/resources/46689 White Rose hub resources – http://glowmathsplanning.com/ks1.html Fluency with fractions rising stars documents – Saved in the common room. No problem text books and workbooks. Shanghai exercise book. <u>Equipment / Models:</u> Bar model – Cuisenaire rods, bar model packs, Thinking blocks website. Magnetic fractions – walls, pizza's, decimal and percentage equivalents Decimal grids showing tenths and hundredths.